

3.7 implicit differentiation

implicit $y + x^2 - x = 0$

explicit $y = -x^2 + x$

ex. $x^2 + y^2 = 1$

explicit

$$y = \pm \sqrt{1 - x^2}$$

implicit
expression

implicit $y^4 + 2xy + y^2 = 7$

$y = -x^2 + x$ find $\frac{dy}{dx}$

$\frac{dy}{dx} = -2x + 1$

$y + x^2 - x = 0$ find $\frac{dy}{dx}$

$\frac{dy}{dx} + 2x - 1 = 0$

$\frac{dy}{dx} = -2x + 1$

implicit diff. process p 159

1. diff. both sides w.r.t. x

2. collect $\frac{dy}{dx}$ terms

3. factor out $\frac{dy}{dx}$

4. solve for $\frac{dy}{dx}$

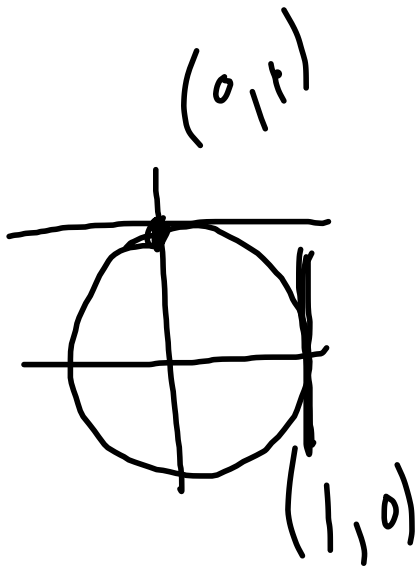
$$x^2 + y^2 = 1$$

↑
in 2 out

$$2x + \underline{2y} \frac{dy}{dx} = 0$$

↑
der of
out

↑
der of in



$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

has x's
& y's
↓

$$(y)^4 + 2xy + (y)^2 = 7$$

find $\frac{dy}{dx}$

$$4y^3 \frac{dy}{dx} + 2x \cdot \frac{dy}{dx} + y \cdot 2 + 2y \frac{dy}{dx} = 0$$

solve
for
 $\frac{dy}{dx}$

$$4y^3 \frac{dy}{dx} + 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2y$$

$$\frac{dy}{dx} (4y^3 + 2x + 2y) = -2y$$

$$\frac{dy}{dx} = \frac{-2y}{4y^3 + 2x + 2y}$$

find y'' if $2x^3 - 3y^2 = 8$

Ex 5

find y' :

$$6x^2 - 6y y' = 0$$

$$\frac{6x^2}{6y} = \frac{6y y'}{6y}$$

$$y' = \frac{x^2}{y}$$

find y''

$$y'' = \frac{y \cdot 2x - x^2 y'}{y^2}$$

$$y'' = \frac{2xy - x^2 \cdot \frac{x^2}{y}}{y^2}$$